MICROCOMPUTER CAPABILITIES - NUMERICAL SIMULATION OF A BREAKING WAVE.

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ABSTRACT. A new microcomputer program using a simple finite differences algorithm has been devised and applied to the simulation of the movement of a water wave generated by a piston-type wavemaker moving at a constant speed in a channel.

A grid is applied on the fluid domain, dividing it into quadrilateral cells where the pressure is constant and in quadrilateral elements obtained by joining the four immediate neighbours of the current node of the grid. During a time increment, an element is considered as a translating solid on which the surface forces are constant along each side. The body forces are constant in the whole element. After having computed the resultant force, Newton's law is applied and the displacement obtained by integrating twice the acceleration, using first order finite differences.

The computing method has been validated by comparison with experimental and numerical results from the literature involving large free surface motion. The crashing of a wave has been successfully simulated. By varying depth and wavemaker speed, it was found that, while the base of the wave has a speed depending only on the depth, the crest has a speed twice that of the wavemaker.The screen of the microcomputer was filmed by a camera triggered by the microcomputer.

1. INTRODUCTION

The movement of water waves has been studied for one century by many authors (Longuet -Higgins (1980), Larras (1979)). The first reported numerical calculations of a crashing wave seems to be those of W.E. Pracht, in Fernbach and Taub(1970), by numerical techniques (Eulerian description and finite differences). Recent papers try to compute the shape of a breaking wave by analytical methods (Greenhow (1983), Peregrine (1983), New et al. (1985), Baker et al. (1982)). With the advent of microcomputers, it may become easier to use simple iterative methods resulting in small programs than analytical methods needing heavy intellectual efforts or sophisticated numerical methods on large computers (Schaeffer (1988). The computing times on microcomputers may still be long, but the gap with large computers is becoming smaller every year. The method described in this paper has been validated on numerical examples from the literature (Welford and Ganaba (1981), Pedersen and Gjevik (1983)) and will be applied here to very large amplitude gravity waves.

2. BASIC EQUATIONS

List of symbols:

c : speed of sound
C : boundary between free surface and inside parts of ∑'
d : diagonal vector of a cell
D : domain of the physical space
dl : line element
ds : surface element
dv : volume element
F : force
g : acceleration of gravity vector
i : index
K : bulk modulus
l : length of a side of a "solid element"
m : mass
n : outer normal unit vector
p : pressure
R : resultant force
t : time
t : tangent unit vector
v : velocity vector
x : position vector
° : initial value
Γ : acceleration vector
∆t :time increment
ρ : specific mass
Ω : domain occupyied by fluid particles
Ω' : domain occupyied by fluid particles having a free surface portion
∑ : boundary of domain Ω
∑' : boundary of domain Ω'

Let us consider (fig. 1) a domain **Ω**, bounded by a surface ∑, occupied by a set of fluid particles that we follow in their movement in a fluid domain ***D*** of the physical space . The fluid contained in **Ω** moves under the action of volume forces (gravity), surface forces (pressure) and line forces (surface tension). If the domain is at the boundary of two different media, like **Ω'**, there is a surface tension along the part of the boundary ∑' which separates the element from the other medium. For small elements we may suppose that the pressure is constant in the element and may neglect rotation. The surface tension, if any, is an external force only at the limit ***C*** between the interior and free surface parts of the frontier ∑' .

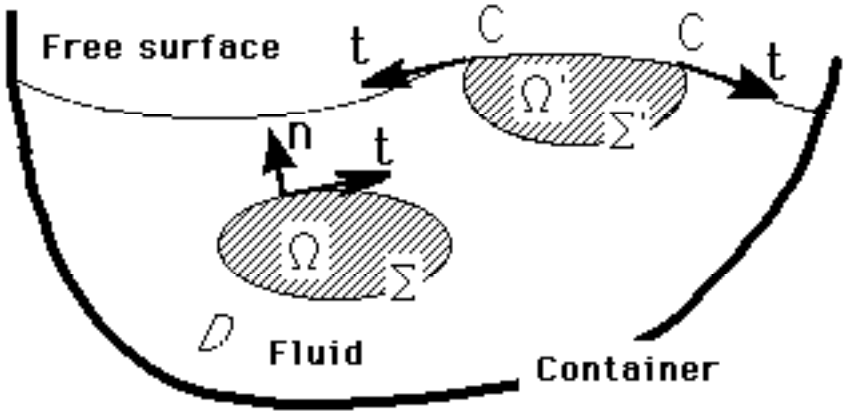


Figure 1. Physical domain of the fluid dynamic problem.

Applying Newton's second law of motion in the integral form, we may write:

$$(1) \qquad \iiint_{\Omega} \rho \mathbf{\Gamma} \, dV = \iiint_{\Omega} \rho \mathbf{g} \, dV - \iint_{\Sigma} p \, \mathbf{n} \, ds + \int_{C} T \, \mathbf{t} \, dl$$

If the acceleration is constant in the fluid domain Ω, equation (1) may be simplifyied:

$$(2) \qquad \Gamma = \mathbf{g} - \frac{1}{m} \iint_{\Sigma} p\,\mathbf{n}\,ds + \frac{1}{m} \int_{C} T\,\mathbf{t}\,dl$$

where

$$(3) \quad m = \iiint_{\Omega} \rho\, dV = \text{constant}$$

From this equation, provided the variables on the right-hand side are known, the new position of the center of gravity is obtained by integrating twice equation (2). For a liquid, the surface tension T is constant and the pressure p is given by the equation of state.

3. NUMERICAL MODEL

The equation (1) of the movement is solved by discretisation of time and fluid space. Time is divided into constant time intervals Δt. A two-dimensional grid (not necessarily rectangular) divides the fluid, assumed to be of unit thickness, in quadrilateral cells in which the pressure is constant. With each node of the grid is associated a "solid element" obtained by joining the four immediate neighbours of the node. This quadrilateral element is considered as a solid moving without rotation during a time increment under the action of the body and surface forces. Hence, the node has the same movement as the center of gravity of the element. A cell is shared by two adjacent elements (fig. 2) and therefore Newton's third law of action and reaction is satisfyied. The difference with classical finite difference or finite elements methods is that the "solid element" is a contour and not a set of nodes.

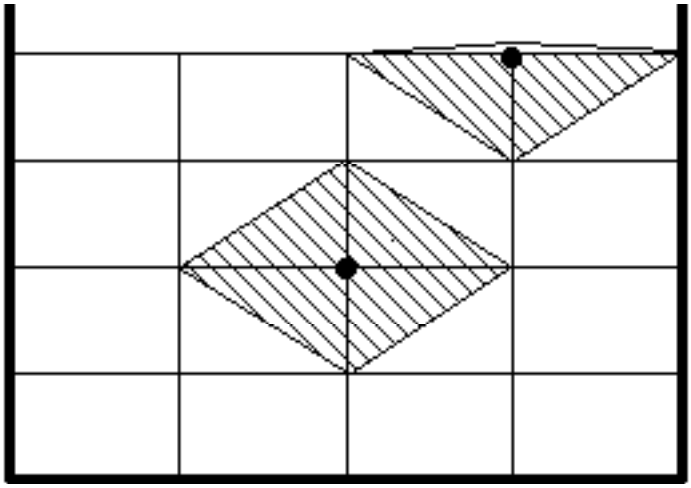


Figure 2. Cells of uniform pressure (rectangular) and "solid" elements (hatched losanges) inside the fluid and at the free surface.

The elements on the boundary are "contracted", that is, the part of the element outside the fluid domain has zero mass, zero volume but prescribed pressure. With this method, it is not necessary to define special elements on the boundary. On the free surface, for each element, the action of the surface tension is described by two forces acting along the two sides of the element coinciding with the free surface.

Figure 3 shows how the continuous medium is divided. The displacements and accelerations are computed on the nodes. A mesh is a quadrilateral of material where the pressure is constant. An element is the contour on which is applied Newton's law, the resultant force being applied to the central node.

The pressure in a cell is proportional to the ratio of the change in area to the initial area of the cell. For a quarilateral, the pressure is simply related to the cross products of the diagonals of the cell:

$$p = -K\left(\frac{\mathbf{d}_1 \times \mathbf{d}_2}{\mathbf{d}^\circ_1 \times \mathbf{d}^\circ_2} - 1\right) \tag{4}$$

Constant pressure is assumed on the free surface. On rigid boundaries, there is free gliding (except at the lower corners) with normal speed equal to

the normal speed of the boundary, if the boundary is moving, zero otherwise. Surface tension is constant in absolute magnitude and acts tangentially to the free surface.

The resultant effort applied on an element is computed by adding vectorially the weight and the forces applied on each of the four sides of the element. These forces $\mathbf{F_i}$ are normal to the sides of the element:

(5) $\mathbf{F_i} = - p_i \mathbf{n_i} l_i$

The resultant force on a solid element is

$$\mathbf{R} = \sum_{i=1}^{4} \mathbf{F}_i \tag{6}$$

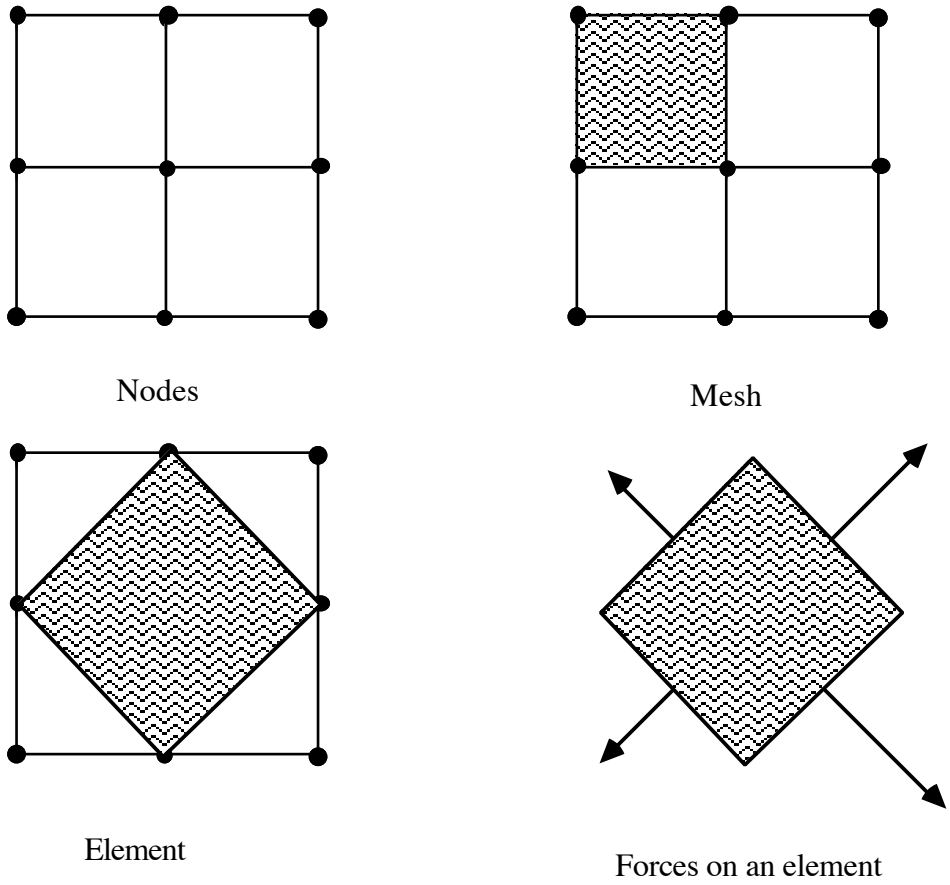


Figure 3. A "solid element" is made from four half meshes.

At each time step, the acceleration, computed by application of Newton's law,

$$\ddot{\mathbf{x}} = \frac{\mathbf{R}}{m} + \mathbf{g} \tag{7}$$

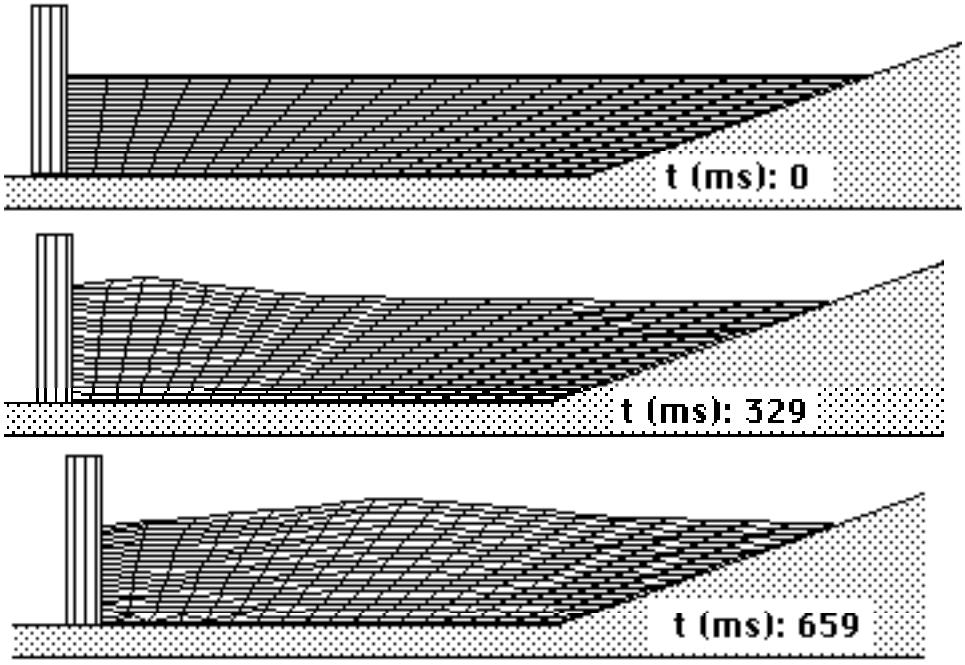
where m is the mass of an element, is integrated numerically twice according to the formulæ:

(8) $\mathbf{V}(t) = \mathbf{V}(t-\Delta t) + \mathbf{\Gamma}\ (t)\ \Delta t$

(9) $\mathbf{X}(t) = \mathbf{X}(t-\Delta t) + \mathbf{V}\ (t)\ \Delta t$

The fluid is initially at rest. At time 0, the external pressure and the gravity are instantaneously applied. It was necessary to use surface tensions much larger than the value for water. The general stability condition was that the Courant number, computed with the speed of sound (not the wave speed) had to be less than one for the smallest cell dimension. In order to minimize the computing time, small values of sound speed were used.

The computation was programmed with the help of UCSD Pascal on an Apple][and later on a Macintosh microcomputer. With the latter, the calculation takes 40 ms per computing cycle and per node instead of 200 ms. On a Macintosh II, if compiled to the arithmetic coprocessor, this time is reduced to a few milliseconds.



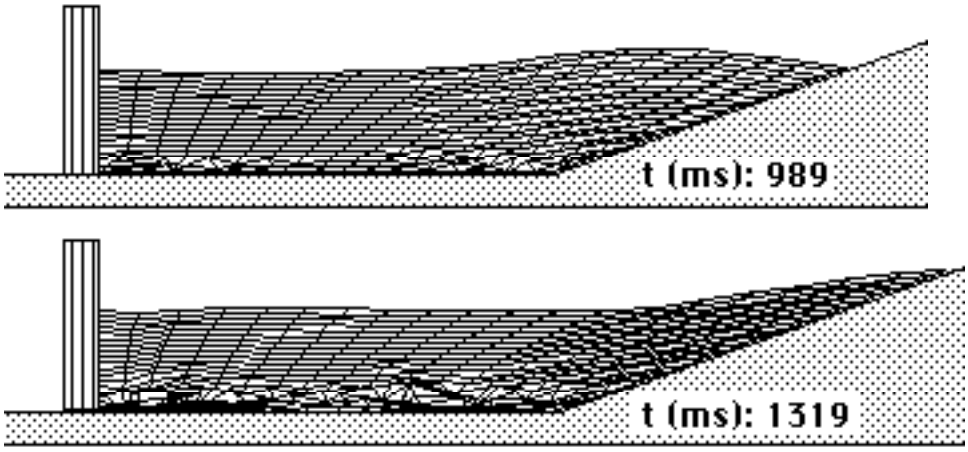


Figure 4. Run-up of a solitary wave generated by the movement of a wavemaker at a speed of 0.2 m/s during 0.5 second with an initial water depth of 0.15 m. and an external pressure of 1 kPa. The speed of sound is 10 m/s, the surface tension is 10 N/m and the specific mass is 1000 kg/m^3 (Schaeffer (1985)).

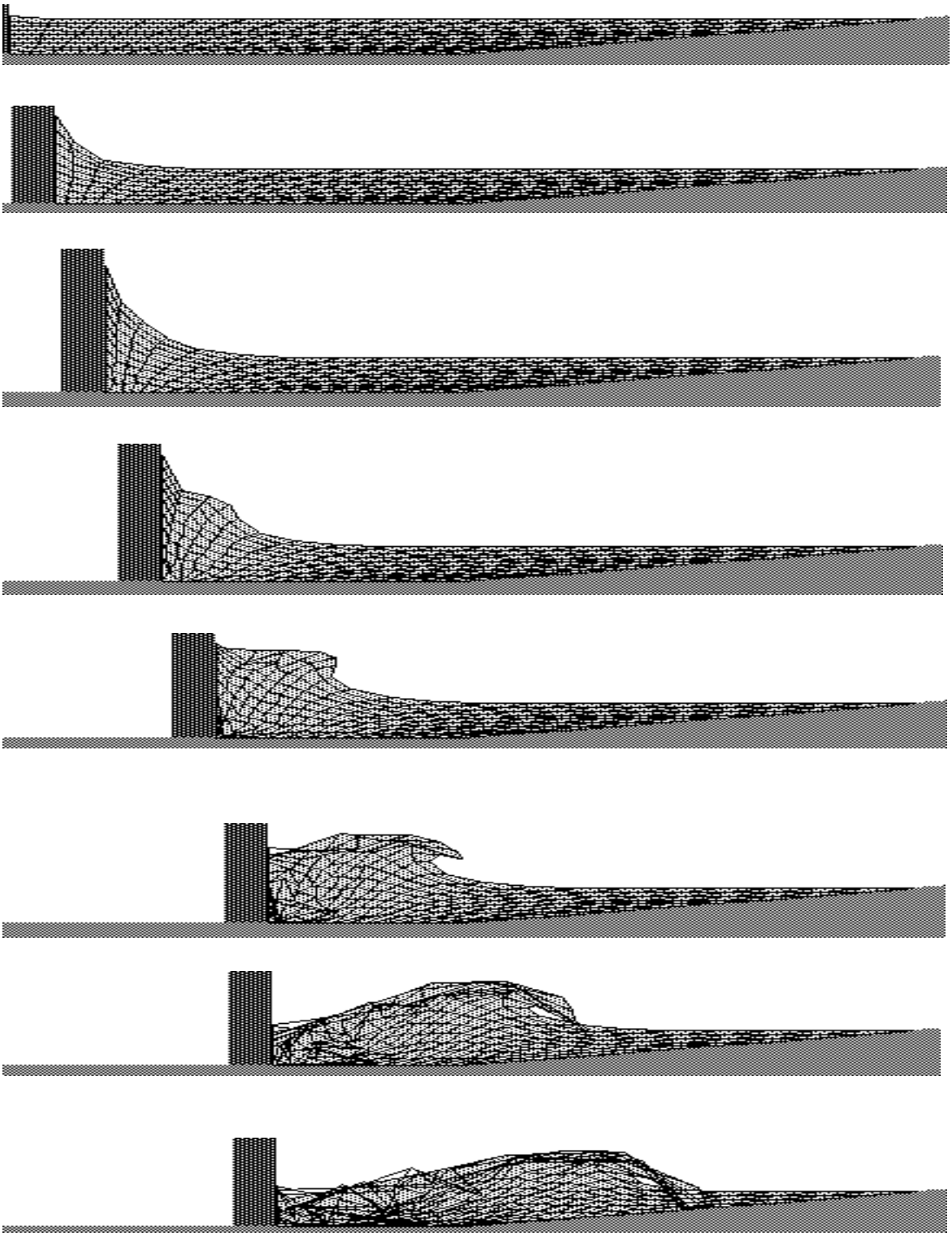


Figure 5. Simulation of a breaking wave. Time sequences of wave overturning with the same conditions as on figure 4 except that the speed of the wavemaker is 1 m/s instead of 0.2 m/s.

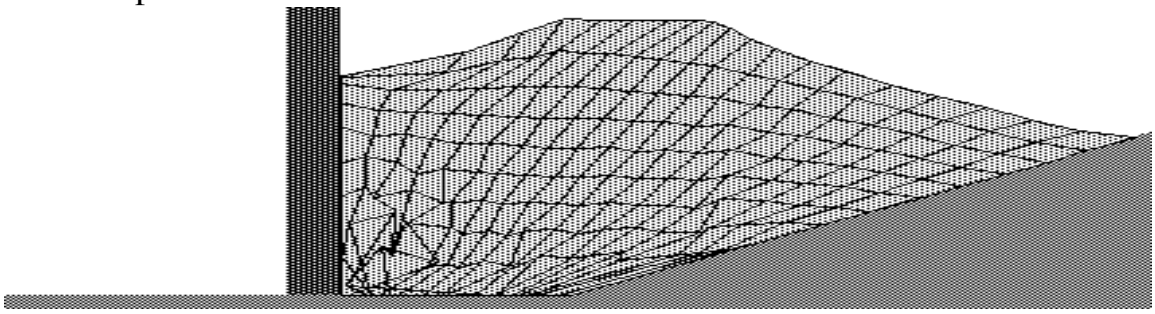
4. Numerical examples

Many experimental and theoretical results of gravity wave propagation may be found in the literature. The configuration studied by Pedersen and Gjevik (1983) numerically and experimentally was chosen to validate the method.

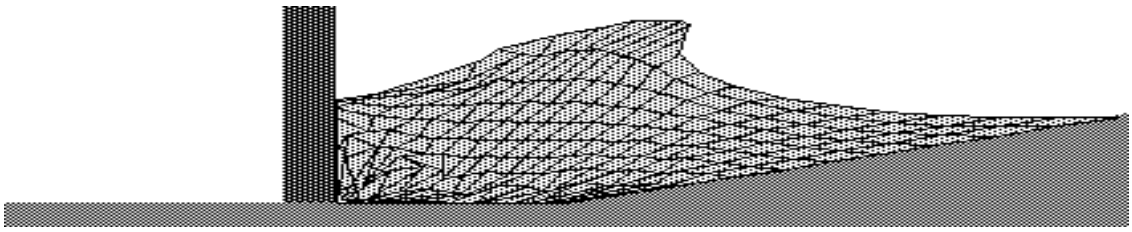
Figure 4 shows the configuration used. The fluid, initially at rest, is put in motion by a wavemaker moving at a constant speed and then stopping. The wave propagates from left to right towards the beach. The figure may be compared directly with the photographs taken by Pedersen and Gjevik (1983). Although the calculation shows numerical "turbulence", the agreement with their experimental results is satisfying. In this particular case, the method does not give better results than theirs except that these calculations are truly bidimensional and therefore allow the simulation of wave overturning.

A second calculation was performed with a speed of the wavemaker approaching the critical wave speed, giving a large amplitude wave (fig. 5). The influence of depth has been visualised on fig. 6, showing calculations for three different depths.Wave breaking occurs when piston speed is less than the critical speed, e.g. when the depth is 0.1 m, as predicted by Lagrange's formula, $v = \sqrt{gh}$. The same has been done on fig. 7, but with varying piston speed. The speed of the bottom of the wave is given by Lagrange's formula. The crest propagates at a speed independant of depth, twice the piston speed.

Initial depth: 0.5 m



Initial depth: 0.2 m



Initial depth: 0.1 m

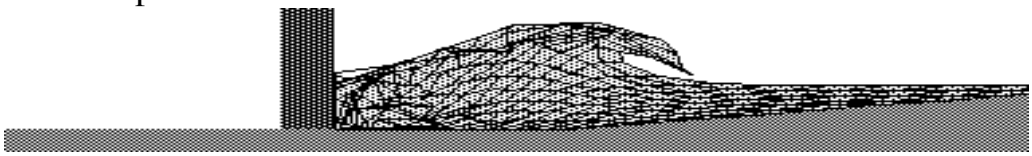


Figure 6. Influence of depth for a piston moving at a constant speed of 1 m/s, at the same moment (0.58 s). The bottom of the wave moves at a speed increasing with depth, but the crest propagates at the same speed, independant of depth.

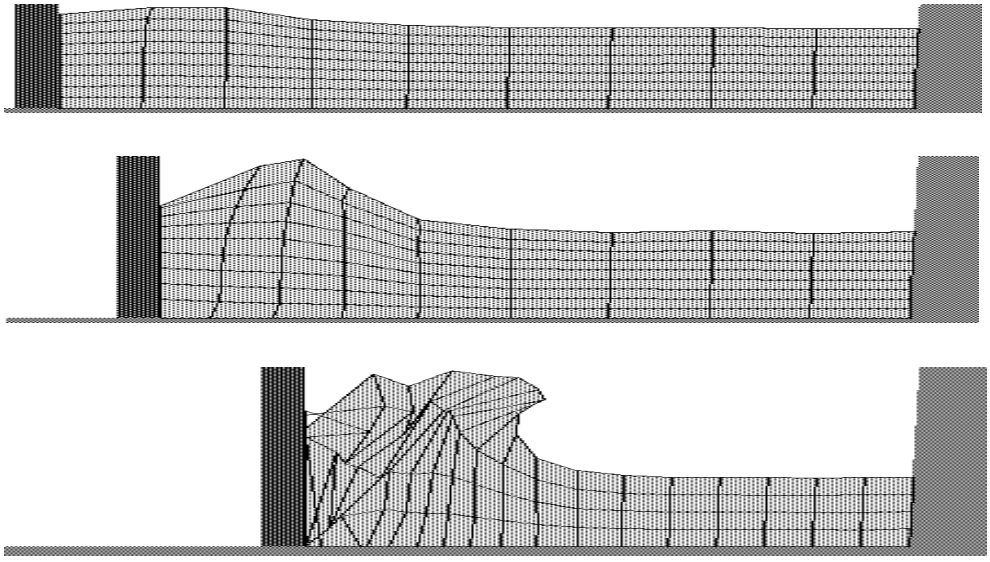


Figure 7. Influence of piston speed (0.1 - 0.5 - 1 m/s) on wave propagation at the same time (t= 0.58 s). Depth: 0.1 m. Wave speed is twice the piston speed.

5. DISCUSSION

The incompressibility hypothesis is usually considered as a simplifying assumption, but it is also possible to use the reverse approximation, that is to replace the slightly compressible fluid by a more compressible fluid, provided that the volume change may be neglected. It will be true if the Mach number is smaller than, for example 0.1.

The main stability criterion is the Courant-Friedrichs-Lewy criterion restricting the distance a wave travels in one time increment to less than one space interval (Hirt (1968)). It applies to the sound speed, not to the gravity wave speed, for a Mach number smaller than one. Numerical instabilities, associated to other criterions, appear after a few thousand iterations.

6. CONCLUSIONS

An explicit Lagrangian prediction algorithm for two-dimensional compressible flow with free surface has been formulated and applied to solitary wave generation, propagation, run-up and overturning. In spite of its simplicity, the method takes into account non-linearity, compressibility, free-surface movement and surface tension. For practical reasons (stability and computing time), the numerical values for the physical properties may be different from those of water, but this has only a slight incidence on the numerical results.

Another advantage of the simplicity of the method its ability to work on microcomputers. It should be useful for the simulation of the two-dimensional problems of fluid mechanics that can be solved in the Lagrangian description.

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